Force and Motion in Our World

Distance and Speed

Measurement and Calculations

Units

Units identify what a specific number represents. For example, the number 42 can be used to represent 42 miles, 42 kilometers, 42 pounds, or 42 elephants. Without the units attached, the number is meaningless. The information is incomplete.

While there are many units systems, we use the SI units (Système International d'Unités).

The metric system (our version of SI) is designed to keep numbers small by converting to similar units by factors of 10.

Prefixes are added in front of a base unit to describe how many factors of 10 the unit has changed.

Base units of measurement are generally described by one letter:

- m – meter (length)
- s – second (time)
- g – gram (mass) *The base unit for mass is actually the kg (kilogram)
- L – litre (volume)

Derived units are combinations of base units. For example, speed is measured in m/s.

Metric Prefixes

See handout of prefixes.

Converting Units

Multiply by One:

- Multiply the measurement by a fraction that equals 1
- The fraction will contain the old unit and the new unit.
- The fraction must cancel out the old unit. (follow the rule that tops and bottoms cancel out)

Ex. Convert 24mm into m.

$$\frac{24\,mm}{1}\left(\frac{1\,m}{1000\,mm}\right)$$

$$\boxed{0.024\,m}$$

Ex. Convert 23 km to cm.

K: 10^3 $3-(-2)=5$

cm: 10^{-2}

$$\frac{23\,Km}{1}\left(\frac{10^5\,cm}{1\,Km}\right) = 2300000\,cm$$

Ex. Convert 15.0 m/s into km/h.

$$\frac{15.0\,m}{s}\left(\frac{1\,Km}{1000\,m}\right)\left(\frac{3600\,s}{1\,h}\right) = 54.0\,Km/h$$

The Sanity Test

After any calculation or problem, stop, take a deep breath, and look at your answer. Does your answer make sense?

Imagine you are calculating the number of people in a classroom. If the answer you got was 1 000 000 people, you would know it was wrong – that's an insane number of people to have in a classroom. That's all a sanity check is: is your answer insane or not?

In using the sanity test, it helps to know typical values of things.

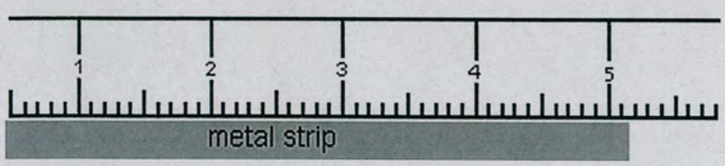
Accuracy vs. Precision

Accuracy is to the extent that a measurement agrees or compares with an accepted value or standard. A very accurate measure of boiling water might be 99.8°C, because it would be compared to the standard of 100°C.

The difference between an observed or measured value and a standard is known as error, and is sometimes written as a percentage.

The accuracy of a measuring instrument depends on how well it compares to an accepted standard, and it should be checked regularly. A known 500.0 g mass should show that same reading on a balance. If it doesn't, the balance should be re-calibrated.

If you were measuring a piece of metal with a ruler (like below), you would get a more exact measurement by using the side graduated in mm (the bottom of the ruler).

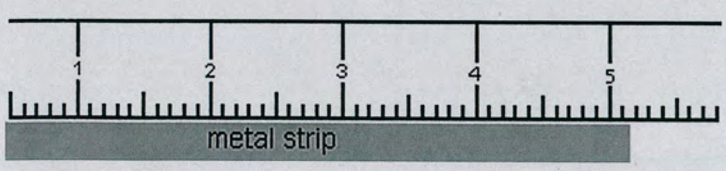


Precision is the degree of exactness that measurement can be reproduced. The precision of a measuring tool is limited by the graduations or divisions on its scale. In other words, you will have a more precise measurement of the metal strip above by using the graduations on the bottom of the ruler (mm) rather than the top (cm).

The precision of an instrument is indicated by the number of decimal places used. For example, 5.14 cm is more precise than 5.1 cm.

Significant Figures

When measuring, the precision is limited by the device - - the number of digits is also limited. Valid digits are called significant digits or significant figures. Significant digits consist of all digits known with a certainty plus the first digit that is uncertain.



The strip above is somewhere between 5.1 and 5.2 cm. We would state that the length is 5.14 cm. The last digit is an estimate (uncertain), but it is still valid and considered to be significant. There are **3 significant digits** in this measurement; the 5 and 1 are known and the 4 is an estimate.

If the strip was dead on the 5.1 graduation, we should record this as 5.10 cm and we would have 3 significant digits.

Rules for Significant Figures

All digits written in scientific notation are significant.

There are 2 rules for determining the number of significant figures.

Decimal Rule

Use this rule when the measurement contains a decimal. Count the numbers from left to right beginning at the first non-zero number.

Ex.

0.001234 4	0.123400 6	1240. 4
1.234 4	12340.0 6	1.234×10^{-3} 4

Non-decimal rule

Use this rule when the measurement **does not** contain a decimal. Count the numbers from right to left beginning at the first non-zero number.

Ex.

1234 4	0.123400 6	12340. 5
102340 5	12340 4	100002 6

Operations with Significant Digits

Adding and Subtracting

The final answer cannot be more precise than the least precise measurement. In other words, the answer must have as few decimal places as the number with the fewest decimal places being added or subtracted.

Ex:

Did Examples

3.414 s + 10.02 s + 58.325 s + 0.000 98 s 71.75998 s $\boxed{71.76 s}$	1884 kg + 0.94 kg + 1.0 kg + 9.778 kg 1895.718 Kg $\boxed{1896 Kg}$
2104.1 m - 463.09 m 1641.01 m $\boxed{1641.0 m}$	2.326 hr - 0.104 08 hr 2.22192 hr $\boxed{2.222 hr}$

Had them try on own

Multiplying and Dividing

Look at the number with the least amount of significant digits. Round the final answer to contain this many significant digits.

Ex.

Did as examples

10.19 cm 4 × 0.013 cm 2 $0.13247 cm^2$ $\boxed{0.13 cm^2}$	140.01 cm 5 × 26.042 cm 5 $\boxed{3646.1 cm^2}$
80.23 m 4 ÷ 2.4 s 2 33.42916667 m/s $\boxed{33 m/s}$	4.301 kg 4 ÷ $1.9 cm^3$ 2 $2.3 Kg/cm^3$

Had them try

Multiple Operations

When a series of calculations is performed, each interim value should not be rounded before carrying out the next calculation. The final answer should then be rounded to the same number of significant digits as contained in the quantity in *the original data* with the lowest number of significant digits.

For example, in calculating (1.23)(4.321) ÷ (3.45 – 3.21), three steps are required:

3.45 – 3.21 = 0.24

(1.123)(4.321) = 5.21483

5.31483 ÷ 0.24 = 22.145125

The answer should be rounded to 22.1 since 3 is the lowest number of significant digits in the original data. The interim values are not used in determining the number of significant digits in the final answer.

Defined Equations

Relationships between variables can be expressed using words, pictures, graphs or mathematical equations. A defined equations is a mathematical expression of the relationship between variables

Ex. Mass and Energy are related by the speed of light

$$E = mc^2$$

Defined equations can be manipulated to solve for any of the variables using the same principles from math.

There are 2 rules that must be followed to isolate a variable:

1) It must be alone
2) It must be on top (numerator)

Ex. Solve $E = mc^2$ for m.

$$\frac{E}{c^2} = \frac{mc^2}{c^2}$$

$$\boxed{\frac{E}{c^2} = m}$$

Ex. Solve $d = \frac{m}{v}$ for v.

$$(v)\, d = \frac{m}{v}(v)$$

$$\frac{vd}{d} = \frac{m}{d}$$

$$\boxed{v = \frac{m}{d}}$$

Relating Speed to Distance and Time

Speed is a rate of change – it is how fast something is moving.

Average speed is total distance travelled divided by the time it took for the trip.

$v_{av} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$	Where v = average speed d_1 = initial position d_2 = final position Δt = total time taken to travel the distance

Δ is the fourth capital letter in the Greek alphabet. It is called "delta" and is read as "change in" and is always the final quantity minus the initial quantity.

For example, $\Delta d = d_2 - d_1$.

Distance is the amount of space between two objects or points. It needs no reference frame. You measure the distance between two objects by measuring their separation. It simply refers to the length (or magnitude) between the two objects. Direction does not matter.

Had them come up with these

Common units for distance:

meters, Kilometers, feet, miles, ~~cubits~~

Time is the duration between two events.

Common units for time:

minutes, seconds, hours

Common units for speed:

miles per hour, m/s, Km/h

Ex. A trip to Calgary is 758 km. If you were to complete the trip in 7.25 h, what was you speed?

$\Delta d = 758\,Km$
$\Delta t = 7.25\,h$
$v = ?$

$v = \frac{\Delta d}{\Delta t}$ $\quad v = \frac{758\,Km}{7.25\,h}$ $\quad v = 104.551724\,Km/h$

$\boxed{v \approx 105\,Km/h}$

Ex. Driving at 95 km/h, how long would it take you to travel 2376 km?

$v = 95\,Km/h$
$\Delta d = 2376\,Km$
$\Delta t = ?$

$(\Delta t)v = \frac{\Delta d}{\Delta t}(\Delta t)$

$\frac{\Delta t\, v}{v} = \frac{\Delta d}{v}$

$\Delta t = \frac{\Delta d}{v}$

$\Delta t = \frac{2376\,Km}{95\,Km/h}$

$\Delta t = 25.01052632\,h$

$\Delta t \approx 25\,h$

Ex. Janna has a summer job helping with bison research. She notes that they graze at an average speed of about 110 m/h for about 7.0 h/day. What distance, in kilometers, will the herd travel in two weeks?

$v = 110\,m/h$
$\Delta d = ?$
$\Delta t = 2wk\left(\frac{7d}{1wk}\right)\left(\frac{7h}{1d}\right) = 98h$

$(\Delta t)v = \frac{\Delta d}{\Delta t}(\Delta t)$

$v\Delta t = \Delta d$

$\Delta d = 110\,m/h\,(98h)$

$\Delta d = 10780\,m\left(\frac{1\,Km}{1000\,m}\right)$

$\Delta d = 10.780\,Km$

$\boxed{\Delta d \approx 11\,Km}$

Instantaneous speed is the speed at which an object is travelling at a particular instant. It is not affected by previous speeds, future speeds, or how long it has been travelling for.

Ex. Looking at speedometer

Constant speed, also called uniform motion, is when the speed of an object remains the same over a period of time. The average speed of an object is the same as its instantaneous speed when it is travelling at a constant speed.

Ex. Cruise control

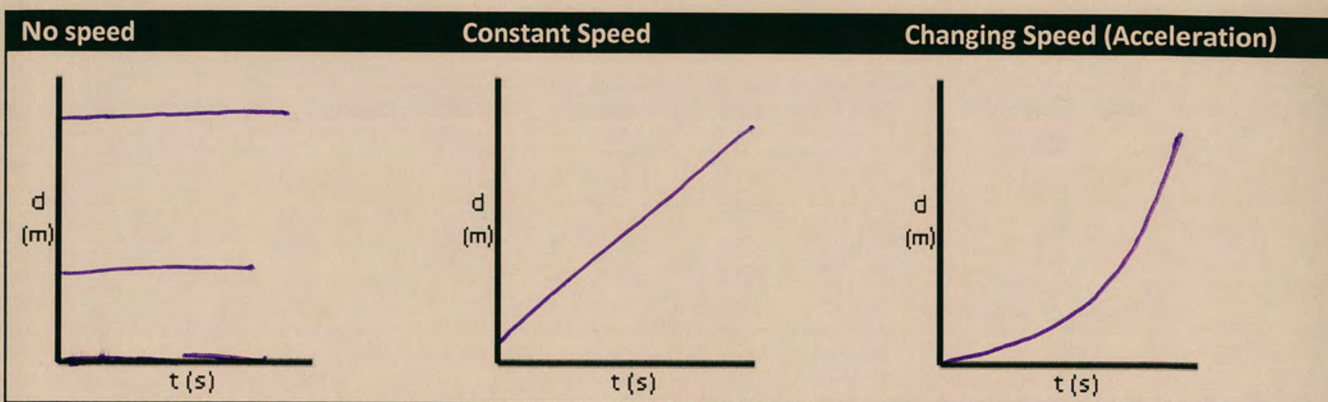
Graphing

See Nelson Science 10 pp. 699 - 701

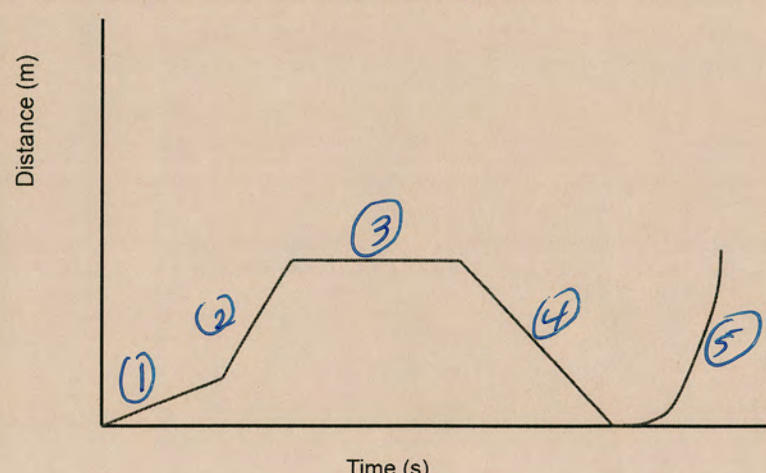
Distance-Time Graphs

We can represent speed with words (fast, slow), numbers (32 km/h) and we can also represent it visually with a graph.

Speed can be represented on a distance vs. time graph. The slope of the graph is the speed. Constant (uniform) motion is given by a straight line on the graph. Curves indicate non-uniform motion. Zero slope represents no motion.



Ex. Describe the motion in the following graph:



1 constant speed

2 constant speed
↳ faster than 1

3 no motion

4 constant motion towards start

5 increasing speed; acceleration

Distance, Speed, and Acceleration

Acceleration

Acceleration is also a rate of change. It is the rate of change in speed and is calculated by the ratio of the change in speed to the time interval that this change happened. It is how quickly something is speeding up or slowing down.

During constant acceleration, the same change in speed happens in each equal interval of time. When acceleration changes over time, we typically describe the object's average acceleration. For our purposes, acceleration is assumed to be constant, so the average acceleration is equal to the constant acceleration.

$a = a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$	Where a = constant acceleration v_1 = initial position v_2 = final position Δt = total time taken to change speed

Tell them not to use these ⟵

Common units for acceleration:

m/s^2 $\quad \frac{m/s}{s} \quad \frac{km/h}{s} \quad \cancel{\frac{km/h}{h}} \quad \cancel{\frac{km}{h^2}}$

Ex. A person on their bike changes their speed from 10.0 m/s to 15.0 m/s in 15.2 s. What is the acceleration of the bike?

$V_1 = 10.0 m/s$
$V_2 = 15.0 m/s$
$\Delta t = 15.2 s$
$a = ?$

$a = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{\Delta t}$

$a = \frac{(15.0 m/s - 10.0 m/s)}{15.2 s}$

$a = 0.3289473684 m/s^2$

$\boxed{a = 0.329 m/s^2}$

↑ can be benefical talking about order of operations on calculators + brackets

Ex. A car is traveling down the road when they see an obstruction. The person accelerates at -3.2 m/s² for 5.0 s until they stop. How fast was the car moving?

$a = -3.2 m/s$
$\Delta t = 5.0 s$
$V_1 = ?$
$V_2 = 0 m/s$

$(\Delta t) a = \frac{\Delta V (\Delta t)}{\Delta t}$

$a \Delta t = \Delta V$

$a \Delta t = V_2 - V_1$

$-V_2 \quad -V_2$

$(-1)(a\Delta t - V_2) = -V_1 (-1)$

$-a\Delta t + V_2 = V_1$

$V_2 - a\Delta t = V_1$

$V_1 = 0 m/s - (-3.2 m/s^2)(5.0 s)$

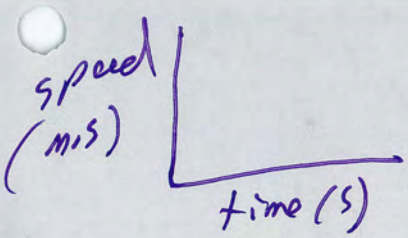
$\boxed{V_1 = 16 m/s}$

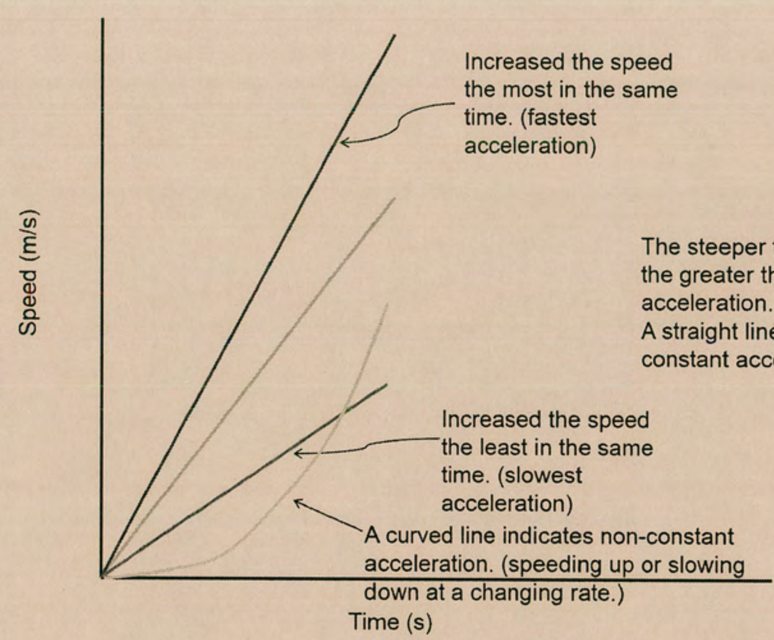
Speed-Time Graphs

We can represent acceleration with words (speeding up, slowing down), numbers (9.8 m/s²) and we can also represent it visually with a graph. This can be done on a Speed-Time graph. On this type of graph, the slope give you the acceleration.

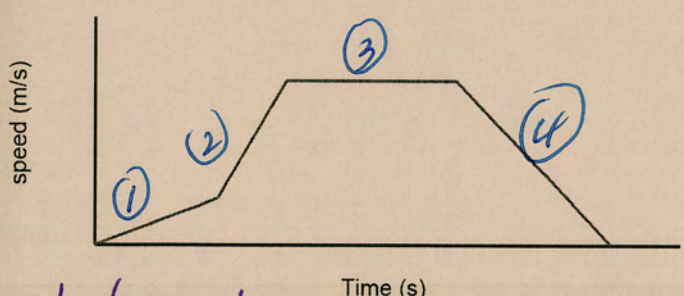
How?

$slope = \frac{\Delta Y}{\Delta X} = \frac{\Delta V}{\Delta t} = a$





Ex. Describe the motion in the following graph.



(1) constant acceleration
→ moderate

(2) constant acceleration
→ greater than (1)

(3) no acceleration → constant speed

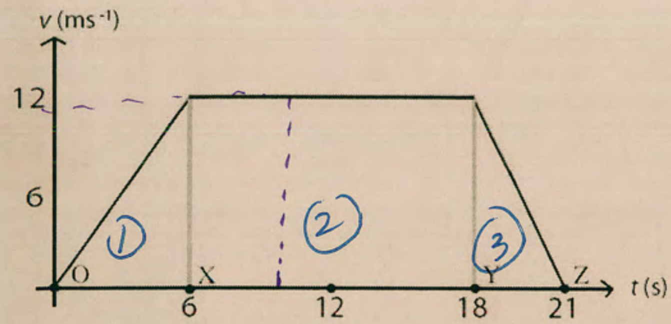
(4) constant acceleration
↳ deceleration

While the slope of the graph gives you acceleration, the area under the curve gives you the distance travelled.

slope | Area

distance
speed
acceleration

Ex. A moped's speed, as it is travelling between two traffic lights, is shown below.



a) When was the moped going a uniform velocity?

6s - 18s

b) When was the moped accelerating? How can you tell?

0-6s → non-zero slope
18-21s

c) When was the moped decelerating? How can you tell?

18-21s → negative non-zero slope ; slowing down

d) How fast was the moped going at 10 s?

12 m/s

e) Calculate the moped's acceleration between 18 and 21 s.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0\,m/s - 12\,m/s}{21s - 18s} = \frac{-12\,m/s}{3s} = \boxed{-4\,m/s^2}$$

f) What was the distance between the traffic lights?

Area under the curve!

1 $\frac{bh}{2} = \frac{6s(12\,m/s)}{2} = 36m$

2 $bh = 12s(12\,m/s) = 144m$

3 $\frac{bh}{2} = \frac{3s(12\,m/s)}{2} = 18m$

Total distance = 1 + 2 + 3 = 36m + 144m + 18m = $\boxed{198m}$

Instantaneous Speed

Instantaneous speed is the speed at a particular moment. Remember that it is unaffected by the speed from earlier or later in the trip.

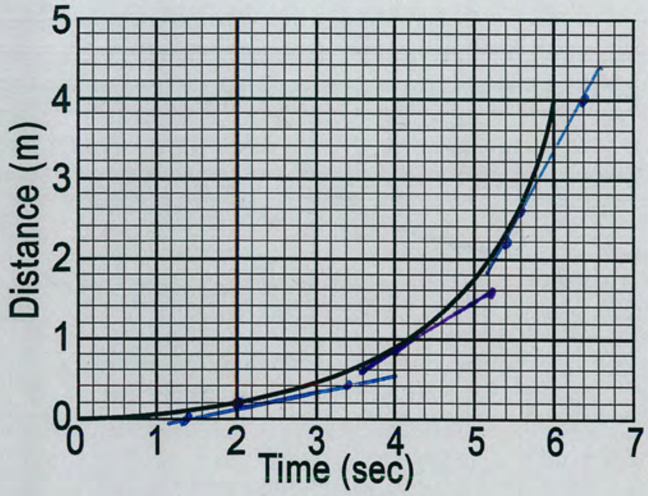
For any object moving at a constant speed, the instantaneous speed is the same at any time and equals the constant speed.

If there is an acceleration, then this is not true, and the distance-time graph will not have a straight line. As such, if we were to take the slope, it would just find us the average speed and not the instantaneous speed.

To find the instantaneous speed with an acceleration present, we have to find the slope of the tangent line to the curve. A tangent is a straight line that just touches the curve at a point and represents the instantaneous slope of the line at that point.

Ex.

★ Make sure they pick points on their tangent lines.



Find the speed of the object at:

a) 2 s

(1.4s, 0m)
(3.4s, 0.4m)

$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.4m - 0m}{3.4s - 1.4s} = \frac{0.4}{2.0}$

$\boxed{v \simeq 0.2\ m/s}$

b) 4 s

(3.6s, 0.6m)
(5.2s, 1.6m)

$\frac{1.6m - 0.6m}{5.2s - 3.6s} = \frac{1.0m}{1.6s}$

$\boxed{v \simeq 0.63\ m/s}$

c) 5.6 s

(5.4s, 2.2m)
(6.4s, 4.0m)

$\frac{4.0m - 2.2m}{6.4s - 5.4s} = \frac{1.8m}{1.0s}$

$\boxed{v \simeq 1.8\ m/s}$

Displacement and Velocity

Frames of Reference

Let's say I am standing on the back of a pickup truck that is motionless and I am throwing apples forward. I know that I can throw an apple at exactly 15 m/s every time. If a person were standing on the sidewalk, how fast would she say the apples are moving? 15 m/s

Now the truck starts moving forwards at 20 m/s. I am still throwing apples forwards, exactly the same as I was throwing them before, at 15 m/s. How fast would the spectator say the apples are moving? 35 m/s

How fast according to me does it look like the apples are moving? 15 m/s

When you are standing on the ground, that is your frame of reference. Anything that you see, watch, or measure will be compared to the reference point of the ground. If I am standing in the back of the moving truck, the truck is now my frame of reference and everything will be measured compared to it. We say that moving objects have relative velocity.

Sitting at your desk, how fast are you moving relative to the ground? 0 m/s Relative to the sun? 30000 m/s Which answer is correct?

We show different motions as arrows in the direction objects are moving. We call these vectors.

Vectors

While motion can be described with words, that is often not good enough. Physics is a mathematical science, so we use two categories of mathematical quantities to describe motion:

Scalars

- quantities described with a magnitude (or number) only
  - Ex. Temperature, mass, and speed

Vectors

- quantities described with both magnitude and direction
  - Ex. Velocity, force, acceleration, displacement, weight, momentum

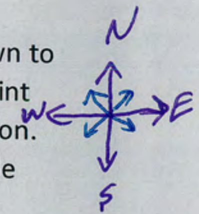
Ex. Which of these is a scalar? A vector?

S V

8 m	26.4 m/s East	84°C	52.0 m/s	3 km [N]
S	V	S	S	V

Drawing Vectors to Scale

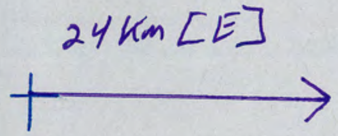
Vectors are used to represent vector quantities on a diagram. A vector is composed of a line segment drawn to scale with an arrowhead at one end. The tail of the vector is at its origin and the head is at the terminal point (the arrowhead). The length of the vector represents its magnitude and the arrowhead indicates its direction. When drawing vectors you must also include reference coordinates. Notice that the direction is given inside square brackets.



Ex. Draw 24 km [E] to scale

1cm = 6 Km

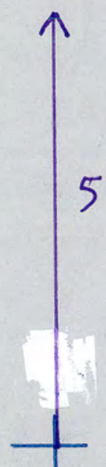
$\frac{24 Km}{6 Km/cm} \Rightarrow 4 cm$



Ex. 57m [N]

1cm = 10m

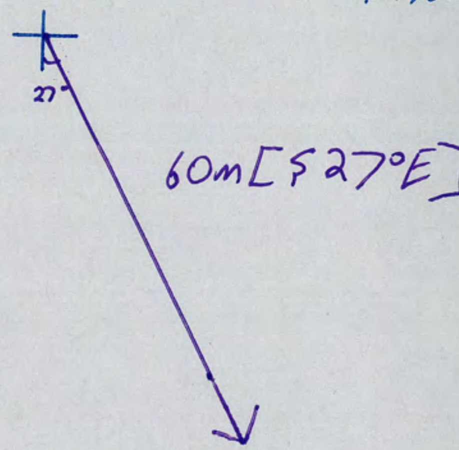
$\frac{57m}{10 m/cm} \Rightarrow 5.7 cm$



Ex. Draw 60 m [S27°E].

1cm = 10 m

$\frac{60m}{10m/cm} \Rightarrow 6cm$



60 m [S27°E] could also be represent by 60 m [27° E of S] or 60 m [297°]. When using the cardinal directions, your angle should be less than 90°.

Collinear vectors are vectors that exist in the same dimension. In other words, they exist either in the same direction or in the opposite direction. Non-collinear vectors are vectors that exist in more than one dimension (i.e. they are located along different straight lines).

Position

Position is the separation and direction from a reference point. It involves both the straight-line distance and direction from the reference point. Usually, the reference point is the origin or starting point. You need to decide which directions will be positive, and which directions are to be negative.

Symbolically, position is shown as $\vec{d}$. Note the arrow to show that it is a vector quantity.

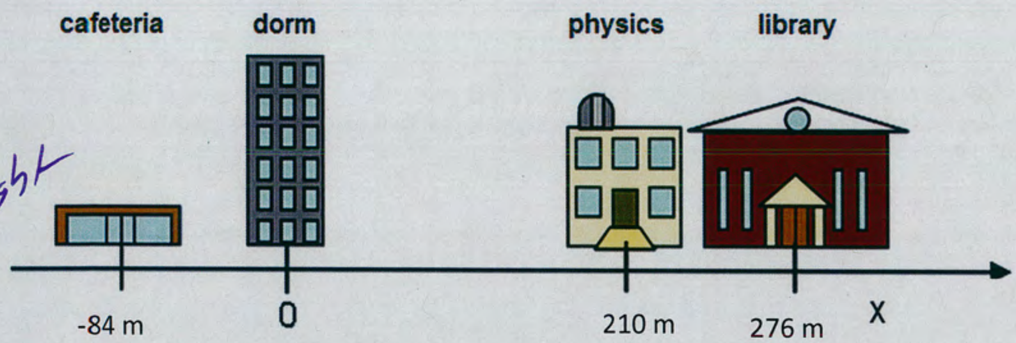
Displacement is your change in position. Symbolically,

$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$	Where $\Delta\vec{d}$ = displacement $\vec{d}_2$ = second position $\vec{d}_1$ = first position

Ex.

Up

Right



Ex. What is your displacement going from the cafeteria to the physics building?

$\vec{d}_1 = -84m$ $\quad \Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$\vec{d}_2 = 210m$ $\quad \Delta\vec{d} = 210m - (-84m) = 294m$ $\quad = \boxed{294m\,[Right]}$

$\Delta\vec{d} = ?$

Ex. What is your displacement moving from the library to the physics building?

$\vec{d}_1 = 276m$ $\quad \Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$\vec{d}_2 = 210\,m$ $\quad \Delta\vec{d} = 210m - 276m = -66m \quad = -66m\,[Right] = \boxed{66m\,[left]}$

$\Delta\vec{d} = ?$

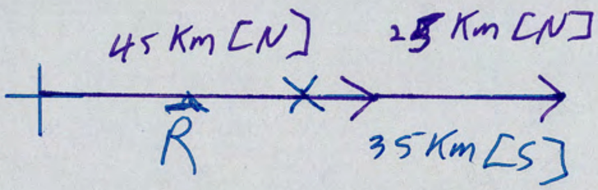
Adding Vectors Along a Straight Line

Using a scale diagram:

1) State the directions
2) List the givens and indicate what variable is being solved
3) State the scale to be used (ex. 1 cm = 5 m)
4) Draw one of the initial vectors to scale
5) Join the second and additional vectors head to tail and to scale
6) Draw and label the resultant vector
7) Measure resultant vector and convert the length using your scale
8) Write a statement including both the size and direction of the resultant vector

Ex. 45 km [N] + 25 km[N] + 35 km [S]

$\rightarrow N$ $\quad 1cm = 10\,Km$



$\vec{R} = 3.5cm$

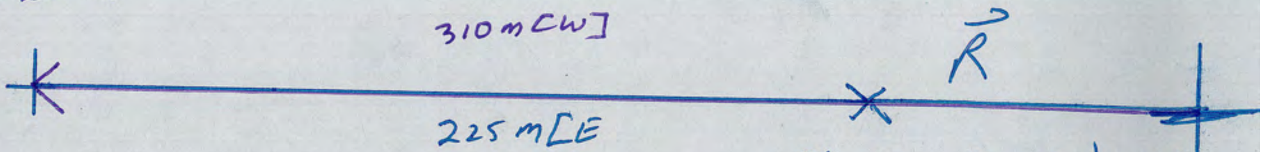
Scale $\quad 3.5cm\,(10Km/cm) = 35Km$ $\quad \boxed{\vec{R} = 35Km\,[N]}$

N
↑→E

$\div 20 = 15.5\text{cm}$ $\div 20 = 11.25\text{cm}$

Ex. John takes his dog, Slatrbartifast the Third, for a walk. They walk 310 m [W] and then back 225 m [E]. Draw a vector diagram to find their resultant displacement.

Let 1cm = 20 m



$\vec{R} = 4.3\text{cm}\left(20\frac{\text{m}}{\text{cm}}\right) = 86\text{m}$

$\boxed{\vec{R} = 86\text{m}[W]}$

Adding vectors algebraically:

To add vectors algebraically, you again have to assign a positive or negative direction to the quantity. At the end of the calculation, you must translate the positive and negative back into a direction.

1) Indicate which direction is positive and which is negative
2) List the givens and indicate what variable is being solved
3) Write the equation for adding the vectors
4) Substitute numbers (with correct signs) into the equation and solve
5) Write a statement with your answer including size and direction

N
↑→E

Ex. Slartbartifast the Third decides to take himself for a walk. He heads 478 ~~k~~m [W], stops, and then goes a further 84 ~~k~~m [W] before returning 243 m [E]. What is his resultant displacement?

$\Delta\vec{d}_1 = 478\,\cancel{k}\text{m}[W] = -478\text{m}$

$\Delta\vec{d}_2 = 84\,\cancel{k}\text{m}[W] = -84\text{Km}$

$\Delta\vec{d}_3 = 243\text{m}[E] = 243\text{m}$

$\Delta\vec{d} = ?$

$\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$

$\Delta\vec{d} = -478\text{m} + (-84\text{Km}) + 243\text{m}$

$\Delta\vec{d} = -319\text{m}$

$\boxed{\Delta\vec{d} = 319\text{m}[W]}$

Adding Vectors at an Angle

This is essentially the same as before, but now the vectors are not all in a line.

1) Use a sharp pencil, ruler and protractor
2) Choose and state a scale that fits your page or space available
3) Calculate the length, to scale, of each of your vectors
4) Draw the compass symbol on your page, with north toward the top
5) Draw the first vector using your ruler and protractor
6) Draw the second vector with its tail at the head of the first
7) Continue adding as many vectors as necessary, always placing the tial of the next vector at the head of the previous arrow
8) Draw the resultant displacement as an arrow from the tail of the first vector (the initial position) to the head of the last vector (the final position)
9) Use your protractor to find its angle from a compass direction and your ruler to measure its length
10) Using your scale, convert the measured length to the actual resultant displacement
11) State the resultant displacement, including size and direction

N
↳E

Ex. Sally walks to Sue's home by going one block west and then two blocks north. Each block is 160 m long. What is Sally's final displacement?

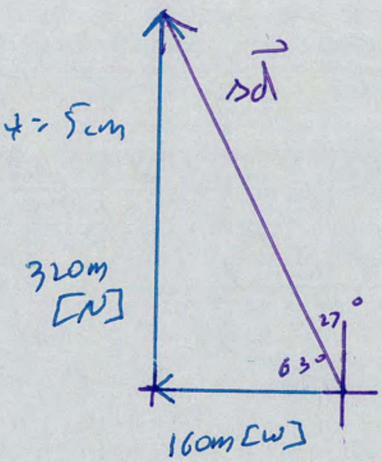
1cm = 64m

$\Delta\vec{d}_1 = 160\text{m [W]} \div 64 = 2.5\text{cm}$

$\Delta\vec{d}_2 = 2(160\text{ m})\text{ [N]} = 320\text{ m [N]} \div 64 = 5\text{cm}$

$\Delta\vec{d} = ?$

$\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$



$\Delta d = 5.55\text{cm}\ (64\text{m/cm})$
$= 355.2\text{ m}$

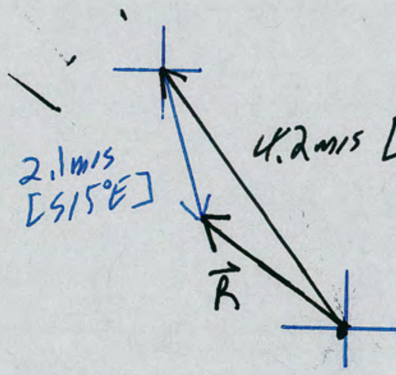
$\Delta\vec{d} = \boxed{355.2\text{m [N27°W]}}$

or

355.2m [W63°N]

Ex. 4.2 m/s [N37°W] + 2.1 m/s [S15°E]

N
↳E 1cm = 1m/s



$\vec{R} = 2.4\text{cm} = \boxed{2.4\text{m/s [N54°W]}}$

Velocity

Velocity is essentially speed along with a direction. Constant velocity means both your speed and direction stay the same.

Average velocity is the overall rate of change of position from start to finish.

$$\vec{V}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

Where $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$
$\Delta t = t_2 - t_1$
$\vec{v}_{av}$ = average velocity

The direction of your average velocity will be in the same direction as the displacement.

Ex. A jogger runs 76 m [E] in 10.0 s and then 51 m [W] for 8.0 s,

→E

a) What is the jogger's velocity for the east portion?

$\Delta\vec{d} = 76\text{m [E]}$

$\Delta t = 10.0\text{ s}$

$\vec{v} = ?$

$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

$\vec{v} = \frac{76\text{m}}{10.0\text{s}}$

$\boxed{\vec{v} = 7.6\text{ m/s [E]}}$

$\rightarrow E$

b) What is the jogger's velocity for the west portion?

$\vec{\Delta d} = 51\,m\,[W] = -51\,m$ $\qquad \vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ $\qquad \vec{v} = \frac{-51\,m}{8.0\,s}$

$\Delta t = 8.0\,s$

$\vec{\Delta v} = ?$

$\vec{v} \approx -6.4\,m/s$

$\boxed{\vec{v} \approx 6.4\,m/s\,[W]}$

$\rightarrow E$

c) What is the jogger's average velocity for the entire run?

$\vec{\Delta d} = 76\,m\,[E] + 51\,m\,[W]$ $\qquad \vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ $\qquad \vec{v} = \frac{25\,m}{18.0\,s}$

$= 76\,m - 51\,m$

$= 25\,m$

$\Delta t = 10.0\,s + 8.0\,s = 18.0\,s$

$\vec{v} = ?$

$\vec{v} \approx 1.4\,m/s$ $\qquad \boxed{\vec{v} \approx 1.4\,m/s\,[E]}$

d) What is the jogger's average speed for the entire run?

$d = 76\,m + 51\,m = 127\,m$ $\qquad \Delta v = \frac{\Delta d}{\Delta t}$ $\qquad \Delta v = \frac{127\,m}{18.0\,s}$

$\Delta t = 18.0\,s$

$\Delta v = ?$

$\boxed{\Delta v \approx 7.06\,m/s}$

Displacement, Velocity and Acceleration

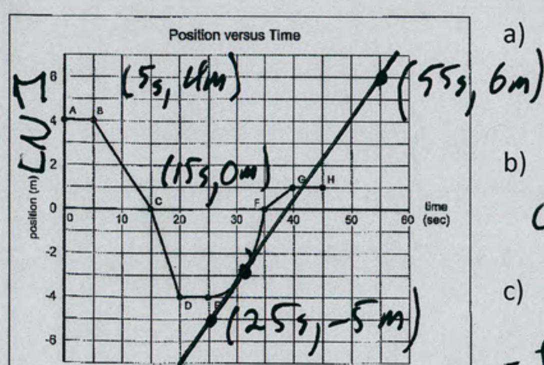
Position-Time Graphs

Position is represented on a position-time graph, which looks very much like a distance-time graph. However, do not forget that position is a vector and will have a direction.

The slope of a position-time graph gives you the velocity of the motion.

The slope of a tangent at a point on a position-time graph gives the instantaneous velocity.

*Add Direction Ex.



a) What is happening from A to B?

No motion.

b) What is happening from B to C?

constant ~~motion~~ velocity south - going back to origin

c) What is happening from C to D?

- faster constant velocity south

d) What is happening at E to F?

- accelerating north

e) Calculate the velocity from B to C.

slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0m - 4m}{15s - 5s} = \frac{-4m}{10s} = -0.4 m/s$

$\boxed{\vec{v} = 0.4 m/s\ [S]}$

f) Find the velocity at 32 s.

-Draw tangent line at 32s

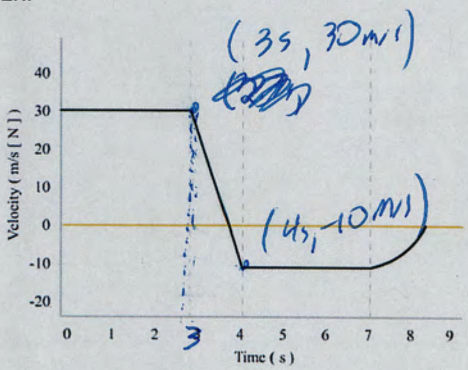
slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6m - (-5m)}{55s - 25s} = \frac{11m}{30s} = 0.37 m/s$

$\boxed{\vec{v} = 0.37 m/s\ [N]}$

Velocity-Time Graphs

On a velocity-time graph, acceleration is given by the slope.

Ex.



What is happening from 0 – 3s?

constant velocity North → 30m/s [N]

Describe the motion from 3s to 4s.

Accelerating South → uniform

Describe the motion from 4s to 7s.

Constant velocity South → 10m/s [S]

What is happening from 7s to 8.5s?

non- uniform acceleration North

What is the acceleration from 3s to 4s?

slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10m/s - 30m/s}{4s - 3s} = \frac{-40m/s}{1s} = -40m/s^2$

$\boxed{\vec{a} = 40 m/s^2\ [S]}$

Average velocity is given by the formula we have already seen, but can also be given by:

$$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Ex. Slartbartifast the Third runs into a wall while taking himself for a walk. What is his average velocity if the initial velocity is 7.4 m/s [S27°W] and he comes to rest 0.17s later? Assume constant acceleration.

$\vec{v}_1 = 7.4 m/s [S27°W]$

$\vec{v}_2 = 0 m/s$

$\Delta t = 0.17s$

$\vec{v}_{av} = ?$

$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2}{2}$

$\vec{v}_{av} = \frac{7.4 m/s [S27°W] + 0 m/s}{2}$

$\boxed{\vec{v}_{av} = 3.7 m/s\ [S27°W]}$

Acceleration and Velocity

Acceleration and velocity are also vector quantities. Our equations are very similar to before:

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

Remember, in your final answer, you must get rid of the positive or negative sign and write in the direction.

$\rightarrow E$

Ex. A train slows down from 50.0 m/s [E] to 34.0 m/s [E] in 4.0 s. What is its acceleration?

$\vec{V}_1 = 50.0 m/s [E]$

$\vec{V}_2 = 34.0 m/s [E]$

$\Delta t = 4.0 s$

$\vec{a} = ?$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

$\vec{a} = \frac{34.0 m/s - 50.0 m/s}{4.0 s}$

$\vec{a} = -4.0 m/s^2 = \boxed{4.0 m/s^2 [W]}$

Ex. Johnny throws a ball straight up from the ground. The ball leaves his hand with an initial velocity of 12.0 m/s. The acceleration of the ball is 9.81 m/s^2 [down].

$\uparrow up$

a) What is the velocity of the ball after 0.50s?

$\vec{V}_1 = 12.0 m/s [up]$

$\vec{a} = 9.81 m/s^2 [down] = -9.81 m/s^2$

$\Delta t = 0.50 s$

$\vec{V}_2 = ?$

$\vec{a} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

$\vec{a} \Delta t = \vec{V}_2 - \vec{V}_1$

$\vec{V}_1 + \vec{a} \Delta t = \vec{V}_2$

$\vec{V}_2 = 12.0 m/s + (-9.81 m/s^2)(0.5$

$\vec{V}_2 = 7.095 m/s$

$\vec{V}_2 \approx \boxed{7.1 m/s [up]}$

b) What is the velocity of the ball after 1.8 s?

$\Delta t = 1.8 s$

$\vec{V}_2 = \vec{V}_1 + \vec{a} \Delta t$

$\vec{V}_2 = 12.0 m/s + (-9.81 m/s^2)(1.8 s)$

$\vec{V}_2 = -5.658 m/s$

$\boxed{\vec{V}_2 \approx 5.7 m/s [down]}$

Handy table to think about:

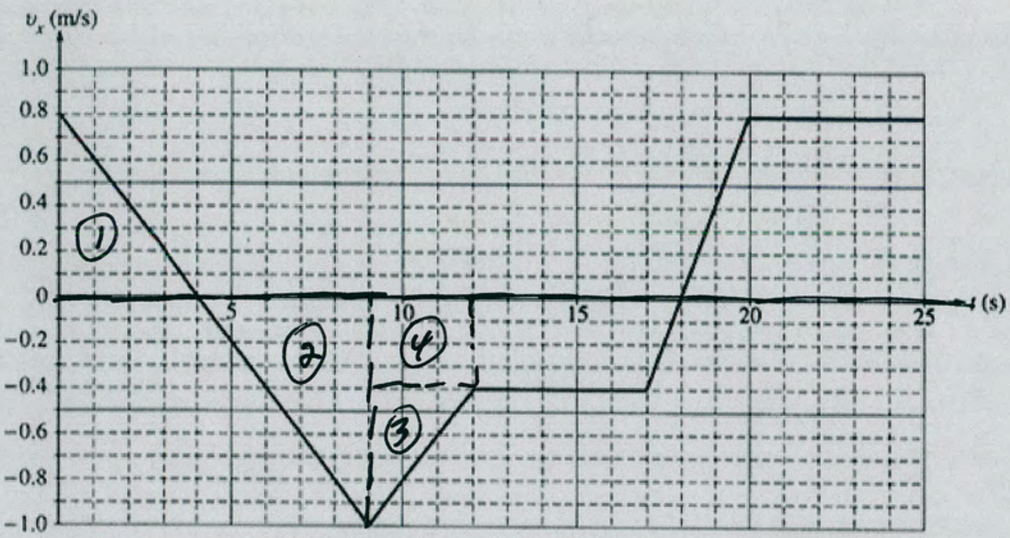
Velocity Direction	Acceleration Direction	Size of Velocity (Speed)
+	+	increase +
-	-	increase −
+	-	decrease +
-	+	decrease −

Displacement from Velocity-Time Graphs

On a Velocity-Time graph, the $\boxed{\text{area}}$ under the curve gives the resultant displacement. Break complex shapes into simple ones and calculate the area. As there will be two directions, be careful of the sign of the area calculated.

Ex. A leaf blowing in a forward direction. Calculate the displacement from 0 s to 12 s.

[N]



① $\dfrac{4s(0.8\,m/s)}{2}$

$= 1.6m$ [N]

② $\dfrac{5s(-1.0\,m/s)}{2}$

$= -2.5m$

$= 2.5m$ [S]

③ $\dfrac{3s(-0.6\,m/s)}{2}$

$= -0.9m$

$= 0.9m$ [S]

④ $3s(-0.4\,m/s)$

$= -1.2m$

$= 1.2m$ [S]

$\Delta\vec{d} = ① + ② + ③ + ④$

$= 1.6m + (-2.5m) + (-0.9m) + (-1.2m)$

$\Delta\vec{d} = -3m$

$\boxed{\Delta\vec{d} = 3m \text{ [S]}}$

Summary for Time Graphs

Slope Area

Position

Velocity

Acceleration

More Equations!

From the graphs, we get that $\Delta\vec{d} = \vec{v}_{av}\Delta t$. Combining this with our previous equation of $\vec{v}_{av} = \frac{\vec{v}_1+\vec{v}_2}{2}$ gives us the following:

$$\Delta\vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$$

This is helpful when you need to find displacement when an acceleration is present, but you do not have the acceleration.

$\uparrow N$

Ex. A cheetah accelerates constantly from 2.3 m/s [N] to 16.7 m/s [N] in a time of about 2.8 s. What displacement happens in that time?

$\vec{V}_1 = 2.3\,m/s\ [N]$

$\vec{V}_2 = 16.7\,m/s\ [N]$

$\Delta t = 2.8\,s$

$\Delta\vec{d} = ?$

$$\Delta\vec{d} = \left(\frac{\vec{V}_1 + \vec{V}_2}{2}\right)\Delta t$$

$$\Delta\vec{d} = \left(\frac{2.3\,m/s + 16.7\,m/s}{2}\right)(2.8\,s)$$

$\Delta\vec{d} = 26.6\,m$

$\boxed{\Delta\vec{d} \approx 27\,m\ [N]}$

By combining $\Delta\vec{d} = \left(\frac{\vec{v}_1+\vec{v}_2}{2}\right)\Delta t$ and $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$, one can get the following:

$$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

* only use this one to solve for Δt if $\vec{V}_1 = 0$

$\rightarrow E$

Ex. A Corvette accelerates constantly from rest at 7.05 m/s [E] for a displacement of 0.0509 km. For how long did this acceleration last?

$\vec{a} = 7.05\,m/s^2\ [E]$

$\Delta\vec{d} = 0.0509\,km = 50.9\,m$

$\vec{V}_1 = 0\,m/s$

$\Delta t = ?$

$$\Delta\vec{d} = \underbrace{\vec{V}_1\Delta t}_{0} + \frac{1}{2}\vec{a}\Delta t^2$$

$$50.9\,m = \cancel{0\,m/s(\Delta t)} + \frac{1}{2}(7.05\,m/s^2)\Delta t^2$$

$$50.9\,m = \frac{1}{2}(7.05\,m/s^2)\Delta t^2$$

$$50.9\,m = 3.525\,m/s^2\,\Delta t^2$$

$$\sqrt{14.43971631\,s^2} = \sqrt{\Delta t^2}$$

$$3.799962672\,s = \Delta t$$

$\boxed{\Delta t \approx 3.80\,s}$

Force

A force is push or a pull. Since these can have different magnitudes and directions, force is a vector quantity. Any force acting on an object can change its shape, its velocity, or both.

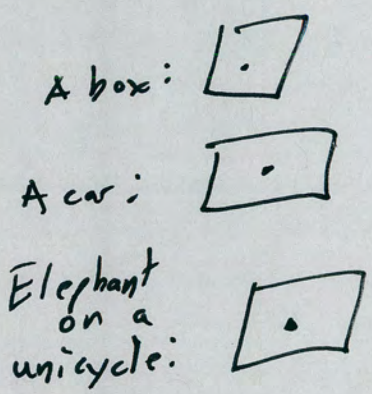
The symbol for force is $\vec{F}$ and the unit is the Newton (N). One Newton is equal to 1 kgm/s^2. Direction is denoted with whatever system is most convenient for the situation.

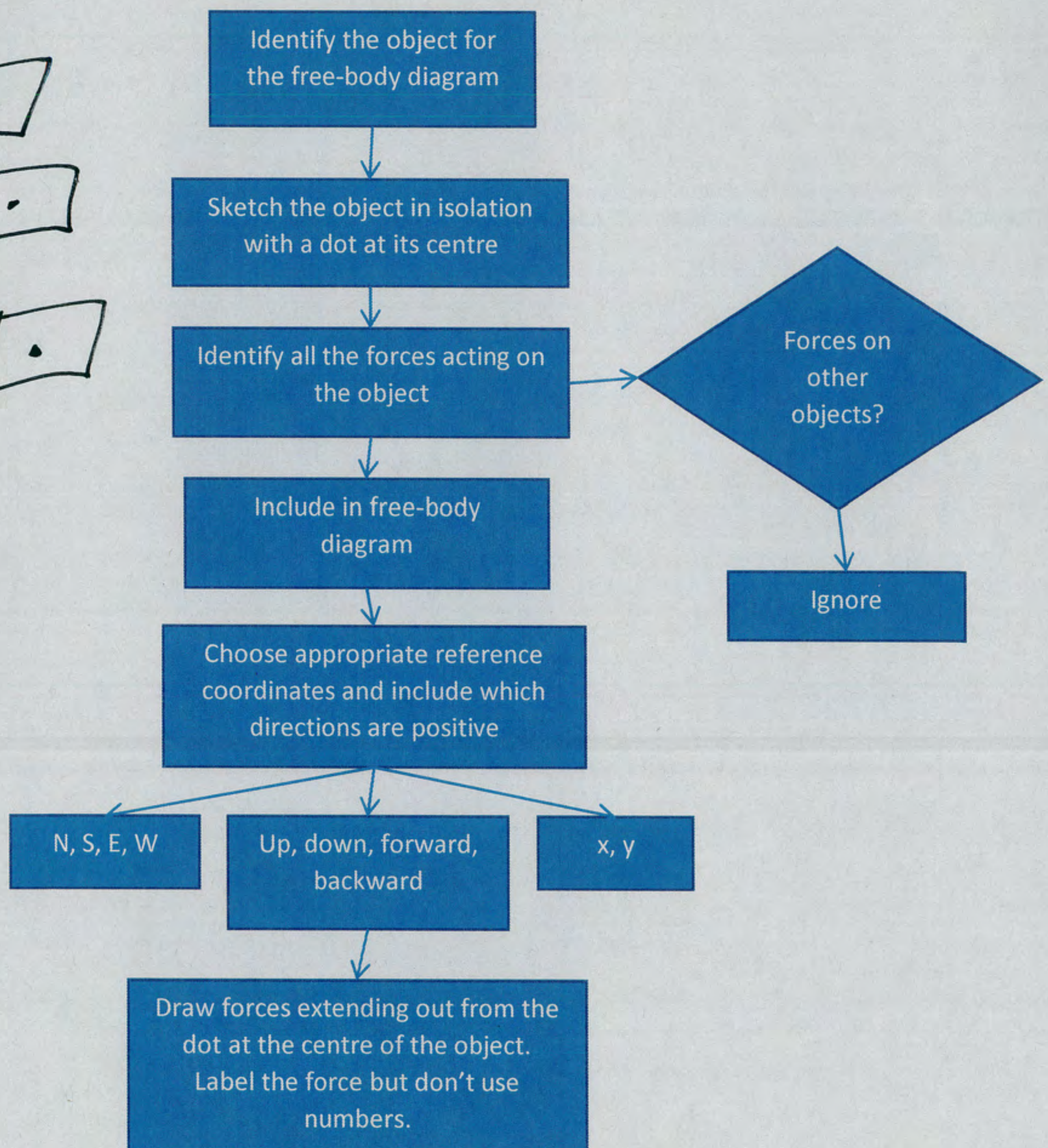
Free-Body Diagrams

Often, there are many forces acting on an object in any given situation and a system is needed to organize these forces. We will use something called a free-body diagram. A free-body diagram is a sketch that shows the object all by itself, isolated from all other objects.

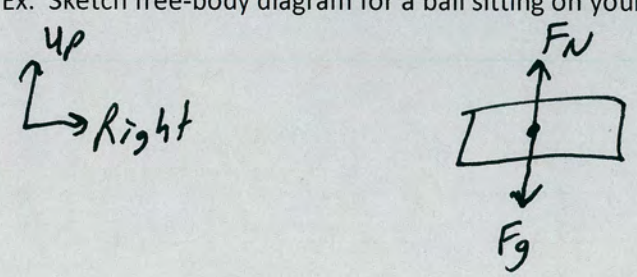
When drawing a free-body diagram, it is important to denote your reference coordinates that are being used.

Flowchart for drawing free-body diagrams:





Ex. Sketch free-body diagram for a ball sitting on your palm.



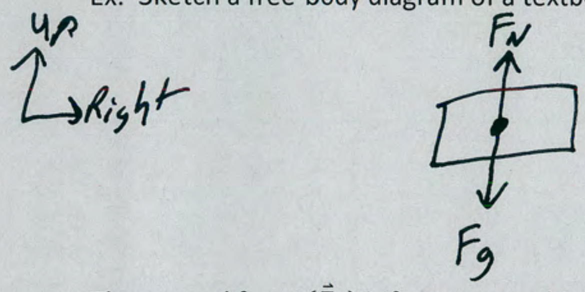
We will see many types of forces acting on an object. Some common that you need to know include:

$\vec{F}_g = force\ due\ to\ gravity$ $\quad$ $\vec{F}_f = force\ due\ to\ friction$ $\quad$ $\vec{F}_N = normal\ force$

$\vec{F}_a = applied\ force$ $\quad$ $\vec{F}_T = force\ of\ tension$ $\quad$ $\vec{F}_{NET} = net\ force$

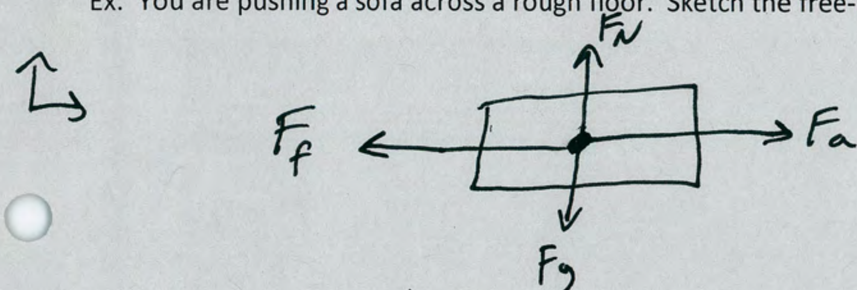
When labeling your diagrams, try to use the same symbols shown above. Capitals and lower case letters do matter.

Ex. Sketch a free-body diagram of a textbook sitting on a table.



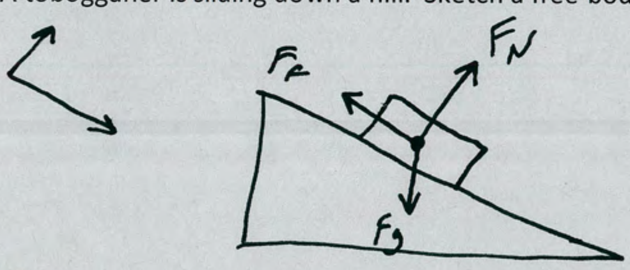
The normal force ($\vec{F}_N$) is force on an object that is perpendicular to the contact surface.

Ex. You are pushing a sofa across a rough floor. Sketch the free-body diagram of the sofa.



The force due to friction ($\vec{F}_f$) always opposes the motion of an object and is parallel to the contact surface.

Ex. A tobogganer is sliding down a hill. Sketch a free-body diagram of the toboggan.



Net Force

The net force acting on an object is the resultant of all forces acting on an object. Equilibrium exists if the net force is zero. We find it by finding the sum of all the forces acting on the object.

$$\vec{F}_{NET} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

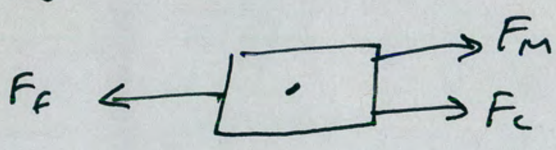
An object in equilibrium will move with a constant velocity.

An unbalanced force exists when the resultant of all the forces acting on an object does not equal zero. If no external unbalanced force acts on an object, its velocity will remain constant. If an unbalanced force exists, the object will begin to accelerate in that direction.

If the forces are parallel to one another, we can just add them together as positive and negative forces. If they are at an angle to one another, we need to add them using a graphical vector diagram created with ruler and protractor.

The unit of force is called the Newton in honour of Sir Isaac Newton. It is equivalent to the kg m/s^2.

Ex. Mr. Birrell's Porsche is stuck in a snow drift. Two enterprising students attach two ropes to the Porsche and attempt to pull it out by pulling in the same direction. Mason pulls with a force of 72 N while Caden pulls with a force of 85 N. There is a force due to friction of 55N acting on the car. Sketch a free-body diagram and determine the net force acting on the car.



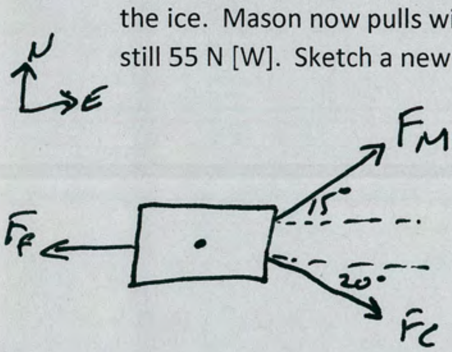
$$\boxed{\vec{F}_{NET} = 102N\ [Right]}$$

$$\vec{F}_{NET} = \vec{F}_M + \vec{F}_C + F_f$$

$$\vec{F}_{NET} = 72N + 85N + (-55N)$$

$$\vec{F}_{NET} = 102N$$

Ex. As Mason and Caden bravely pull the car, they notice a patch of ice on the road directly in front of them. To keep on pulling without wiping out on the ice, they must begin to pull at an angle as they walk around the ice. Mason now pulls with 72 N [E15°N] and Caden pulls with 85 N [E20°S]. The force due to friction is still 55 N [W]. Sketch a new free-body diagram and determine the new net force.



$\vec{F}_f = 55N\ [W]$ $\div 15 \approx 3.7$

$\vec{F}_M = 72N\ [E15°N]$ $\div 15 \approx 4.8$

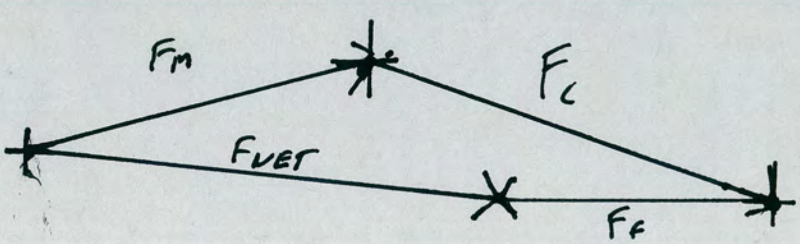
$\vec{F}_C = 85N\ [E20°S]$ $\div 15 \approx 5.7$

$$\vec{F}_{NET} = \vec{F}_M + \vec{F}_C + \vec{F}_f$$

$1cm = 15N$

$F_{NET} = 6.3cm\ (15) = 94.5N$

$\vec{F}_{NET} = 94.5N\ [S83.5°E]$



Acceleration Due to Gravity

Why do things fall?

- force of gravity

Acceleration due to gravity affects objects falling towards other massive objects, such as a planet.

To solve equations involving the acceleration due to gravity, any of the equations of motion we have used this unit may work if they involved an acceleration. We will assume that the acceleration due to gravity, $\vec{g}$, is equal to 9.81 m/s^2 [down] unless otherwise specified.

↑up Ex. A jackalope jumps up into the air. As he leaves the ground he is travelling at 16.0 m/s. How fast is he going after 2.8 s?

$\vec{v}_1 = 16.0\,m/s\,[up]$

$\Delta t = 2.8\,s$

$\vec{v}_2 = ?$

$\vec{a} = 9.81\,m/s^2\,[down]$

$= -9.81\,m/s^2$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\vec{v}_2 = 16.0\,m/s + (-9.81\,m/s^2)(2.8\,s)$

$\vec{v}_2 = -11.468\,m/s$

$\vec{v}_2 \approx \boxed{11\,m/s\,[down]}$

Ex. A baseball is dropped from a height of 1.238 m at the North Pole. Repeated trials showed that the average time for the baseball to fall from rest was 0.52 s. Calculate the acceleration due to gravity at the North Pole.

↑up

$\Delta\vec{d} = 1.238\,m\,[down]$

$= -1.238\,m$

$\Delta t = 0.52\,s$

$\vec{v}_1 = 0\,m/s$

$\vec{a} = ?$

$\Delta\vec{d} = \cancel{\vec{v}_1\Delta t}^{\,0} + \frac{1}{2}\vec{a}\Delta t^2$

$2(\Delta\vec{d}) = (\frac{1}{2}\vec{a}\Delta t^2)2$

$\frac{2\Delta\vec{d}}{\Delta t^2} = \frac{\vec{a}\Delta t^2}{\Delta t^2}$

$\frac{2\Delta d}{\Delta t^2} = \vec{a}$

$\vec{a} = \frac{2(-1.238\,m)}{0.52\,s}$

$\vec{a} = -9.156804734\,m/s^2$

$\vec{a} \approx -9.2\,m/s^2$

$\boxed{\vec{a} \approx 9.2\,m/s^2\,[down]}$